PG – 206

II Semester M.Sc. Degree Examination, July 2017
(CBCS)
MATHEMATICS
M 203 T : Topology – II

Time : 3 Hours　　　　　　　　　　　　　　　　Max. Marks : 70

*Instructions : i) Answer **any five** full questions.*
*ii) **All** questions carry **equal** marks.*

1. a) Prove that a continuous image of a compact space is compact and also show that an infinite set X with the co-finite topology is compact.

   b) Every countable open cover of $(X, \mathcal{T})$ has a finite subcover. Then prove that $(X, \mathcal{T})$ is countably compact.

   c) Prove that every closed subset of a locally compact space is locally compact. **(7+4+3)**

2. a) Prove that second countability is both topological and hereditary property.

   b) Define a separable space. Prove that a metric space which is Lindeloff is a second countable.

   c) Prove that a metric space (X, d) is countably compact iff every countable open cover has a finite subcover. **(5+5+4)**

3. a) Show that a mapping $f : Z \to X \times Y$ is continuous iff Π_X of and Π_Y of are continuous.

   b) Show that $X \times Y$ is first countable iff X and Y are first countable.

   c) Prove that if A is closed in $(X, \mathcal{T})$ and B is closed in $(Y, \mathcal{T})$ then $A \times B$ is closed in the product topology and conversely. **(4+4+6)**

4. a) Prove that in a T_0-space the closure of distinct points are distinct and conversely.

   b) Define a T_1-space. Prove that T_1 property is topological.

   c) Show that a compact subset of a Hausdorff space is closed. **(5+4+5)**

P.T.O.

5. a) Show that X is regular if and only if for every open set G containing x, there exists an open set G^* such that $x \in G^* \subseteq \bar{G}^* \subseteq G$.

b) Prove that a regular space need not be a T_1-space.

c) Define T_3-space. Prove that every metric space is a T_3-space. (5+4+5)

6. a) Define a normal space. Is normality a hereditary property ? Justify your answer.

b) Give an example for a T_2-space need not be a normal.

c) Show that a regular Lindeloff space is normal. (4+3+7)

7. a) State and prove Tietze's extension theorem.

b) Show that normal space is regular if and only if it is completely regular. (10+4)

8. a) Prove that a complete normality is a hereditary and topological property.

b) Show that the set of reals with the discrete topology is completely normal but not when the topology is the co-finite topology.

c) Prove that every paracompact Hausdorff space is normal. (6+4+4)

Completely normal